WM-04-108
February 2, 2008

FCNC and Rare B Decays in 3-3-1 Models

J.-Alexis Rodriguez[1]

Departmento de Fisica

Universidad Nacional de Colombia, Bogota, Colombia

and

Marc Sher[2]

Particle Theory Group

Department of Physics

College of William and Mary, Williamsburg, VA 23187, USA

An interesting extension of the Standard Model is based on the electroweak gauge group $SU(3)_L \times U(1)$. It requires three generations to cancel anomalies, treats the third generation differently than the first two, and has a rich phenomenology. There are several models, distinguished by the embedding of the charge operator into the $SU(3)_L$ group and by the choice of fermion representations. In this Brief Report, we consider flavor-changing neutral currents in these models, concentrating on the $P-\overline{P}$ mass difference, where $P=(K,D,B,B_s)$, as well as $B \to Kl^+l^-$, $B \to \mu^+\mu^-$ and $B_s \to \mu^+\mu^-$ decays. Although the $P-\overline{P}$ mass difference has been considered previously in some models, the rare B decays are new. We find that the strongest bounds come from the $B-\overline{B}$ and $B_s-\overline{B}_s$ mass difference.

[1]Email: jarodriguezl@unal.edu.co
[2]Email: sher@physics.wm.edu

1 Introduction

One of the more intriguing extensions of the standard model is based on the gauge group $SU(3)_c \times SU(3)_L \times U(1)$. In the original, minimal version of the model[1, 2], the charged leptons and neutrinos are put into antitriplets of $SU(3)_L$, two generations of left-handed quarks are put into triplets and the other generation into an antitriplet. This structure automatically cancels all anomalies, and when combined with the requirement of asymptotic freedom, necessitates that the number of generations is equal to three. The model has an automatic Peccei-Quinn symmetry[3, 4]. The fact that one of the quark families is treated differently than the other two could lead to an explanation of the heavy top quark mass[5]. This minimal model contains doubly charged bilepton gauge fields, as well as isosinglet quarks with exotic charges, leading to a rich phenomenolgy[6]. A particularly exciting feature of this model is that there is an *upper* bound on the scale of $SU(3)_L$ breaking which is within range of the LHC.

In another version of the model, with a different embedding of the charge operator into $SU(3)_L \times U(1)$, the charged lepton in the antitriplet is replaced by a right-handed neutrino[7, 8]. In this version, the bileptons are singly charged or neutral. Another model can be found in which there are no lepton-number violating gauge bosons and no exotic quark charges (at the price of adding an isosinglet charged lepton for each generation). Nonetheless, in all of these models, one still treats one of the quark generations differently than the other two.

It is most natural to have the third generation be the "different" generation, since this might explain the heavy top quark and since some of the constraints to be discussed below are substantially weakened. With generations treated differently, one will expect to have tree-level flavor-changing neutral currents (FCNC). Thus, it is expected that FCNC involving the third generation will be dominant. Given the success of BELLE and BABAR, an analysis (and update of previous analyses) of rare B decays and FCNC in these models seems warranted.

In the next section, we discuss the three models mentioned above, as well as two other

models in which all of the generations are treated identically. In section III, we analyze current bounds from FCNC processes in these models. Section IV contains our conclusions.

2 Models

A comprehensive review of the gauge, fermion and scalar sectors of the various $SU(3)_L \times U(1)$ models can be found in Refs. [9] and [10]. In this section, we briefly summarize this review, and then turn to a discussion of FCNC and rare B decays in these models. Different models can be distinguished by the embedding of the electric charge operator. In general, the charge operator is given by

$$Q = aT_{3L} + \frac{2}{\sqrt{3}} bT_{8L} + xI_3, \tag{1}$$

where we have used conventional normalization ($T_i = \lambda_i/2$ and $\mathrm{Tr}(\lambda_i\lambda_j) = 2\delta_{ij}$), I_3 is the 3x3 unit matrix, and a and b are arbitrary. The value of x can be absorbed into the hypercharge definition, and will not be relevant. The fact that weak isospin is contained within the $SU(3)_L$ group implies that $a = 1$, and so models are distinguished by the value of b. It should be noted that gauge bosons will have integral charge only for half-integral values of b, and that models with negative b can be transformed into models with positive b by replacing triplet fermion representations with antitriplets, and vice versa.

The two choices for b that have been considered are $b = 3/2$ and $b = 1/2$. The former gives the original, minimal Pisano-Pleitez-Frampton model, with exotic isosinglet quark charges, while the latter does not lead to any exotic quark charges. We now discuss each choice.

Of the 9 gauge bosons of the electroweak group, 3 are neutral and there are three charged pairs, the usual $W^{\pm}$ and two others with charges $\pm(b+1/2)$ and $\pm(b-1/2)$. Thus this $b = 3/2$ model has doubly charged gauge bosons. In the minimal model, the fermion representations are

$$L_i = \begin{pmatrix} e \\ \nu_e \\ e^c \end{pmatrix}, \begin{pmatrix} \mu \\ \nu_\mu \\ \mu^c \end{pmatrix}, \begin{pmatrix} \tau \\ \nu_\tau \\ \tau^c \end{pmatrix} \tag{2}$$

for the leptons, and

$$Q_i = \begin{pmatrix} u \\ d \\ D \end{pmatrix}, \begin{pmatrix} c \\ s \\ S \end{pmatrix}, \begin{pmatrix} b \\ t \\ T \end{pmatrix} \tag{3}$$

for the left-handed quarks. The conjugates for these nine fields are all $SU(3)_L$ singlets. D and S are new isosinglet quarks with charge $-4/3$ and T is an isosinglet quark with charge $5/3$. Note how the third generation is treated very differently than the first two. This is necessary to cancel anomalies. In principle, either of the three generations could be chosen to be different, however, as will be seen shortly, the strong bounds on FCNC in the kaon sector make it more likely that the third generation is singled out. This is the original, minimal model, and will be referred to as Model A.

When an extension of the standard model predicts new phenomena, one can often explain non-observation of the phenomena by increasing the mass scale of the new physics. However, that is not possible for the minimal $SU(3)_L \times U(1)$ model. The reason is that if one were to embed the standard model entirely into the $SU(3)_L$ group, then the unification gives $\sin^2\theta_W = 1/4$. The extra $U(1)$ factor then forces one to have $\sin^2\theta_W \leq 1/4$. This is, of course, valid at low energy, but since $\sin^2\theta_W$ increases with scale, the scale of $SU(3)_L$ breaking cannot be too high. In the original, minimal model, model A, this scale was estimated to be approximately 800 GeV. It has been argued[11] that more precise definitions of "scale" allow this upper bound to be somewhat higher, as high as 2-3 TeV. Thus, the model is capable of being ruled out in the near future.

A simple alternative to this model[12] is to change the lepton structure by replacing the e^c_i with a heavy lepton E^+_i and adding e^c_i and E^-_i singlets. This will be referred to as Model A'.

Although one can, of course, add a right-handed neutrino singlet to the above structure, the model of Montero, et al.[7, 8] modifies the lepton sector, and has, with $b = -1/2$,

$$L_i = \begin{pmatrix} \nu_i \\ e_i \\ \nu^c_i \end{pmatrix} \tag{4}$$

with the e_i^c being an $SU(3)_L$ singlet. The quarks are given by

$$Q_i = \begin{pmatrix} d \\ u \\ D \end{pmatrix}, \begin{pmatrix} s \\ c \\ S \end{pmatrix}, \begin{pmatrix} t \\ b \\ T \end{pmatrix} \tag{5}$$

The new weak isosinglet quarks now have the same charges as their standard model counterparts, and the bileptons are either neutral or singly-charged. This model will be referred to as Model B.

Since additional exotic quarks must be introduced in these models, it is natural, in the spirit of grand unification, to suppose that additional charged leptons are present. In Model C, the leptons are taken to be

$$L_i = \begin{pmatrix} \nu_i \\ e_i \\ E_i \end{pmatrix} \tag{6}$$

and the quarks are

$$Q_i = \begin{pmatrix} d \\ u \\ U \end{pmatrix}, \begin{pmatrix} s \\ c \\ C \end{pmatrix}, \begin{pmatrix} t \\ b \\ B \end{pmatrix} \tag{7}$$

with all other fields (including right handed neutrinos, if necessary) being $SU(3)_L$ singlets. This model has been explored in Ref. [13]

In all of the above models, the quark generations are treated differently. There are two other models[9, 10] with identical quark generations to the previous two models, but in which the leptons are all treated very differently. These models have not been explored in detail, and since we are interested in FCNC in the quark sector, they will not be discussed further here.

Finally, there are two models in which all generations, quark and leptons are treated equally. These models lose the appealing feature of explaining the number of generations (via anomaly cancellation), but do have the feature of following naturally from grand unified theories. In each of these models, there are 27 fields in each generation. In Model D, these fields fill out a 27 of E_6, and arises naturally from the E_6 GUT. This model has been analyzed in Ref. [14]. Model E has a "flipped" structure, and arises from an $SU(6) \times U(1)$ unified gauge symmetry, and has been discussed in Ref. [15].

Nothing in the above discussion is new, and there has been some phenomenological work on all of these models. However, there has been very little done (especially in the three-generation models A,B and C) regarding FCNC B-decays, and the bounds from Δm_B and ΔM_{B_s} need to be updated. We turn to these issues in the next section.

It should be pointed out that the scalar sector of these models all contain at least three $SU(3)_L$ triplets[16], and in some cases an additional Higgs sextet is needed to give leptons mass[17]. These Higgs triplets may give additional contributions to FCNC processes. However, since these contributions will depend on large numbers of arbitrary parameters, we will ignore them–their inclusion would only strengthen the lower bounds on gauge boson masses (unless they interfere destructively and one fine-tunes).

3 FCNC and rare B decays

With different generations treated differently, it is not surprising that tree level flavor-changing neutral currents will arise. A nice discussion of FCNC interactions in the minimal model, model A, can be found in the works of Liu[19] and Gomez Dumm, et al[20]. They show that

$$\mathcal{L}_{FCNC} = \frac{g}{\cos\theta_W}\frac{1}{2\sqrt{3}\sqrt{1-4\sin^2\theta_W}}(-\sin\phi Z_{1\mu} + \cos\phi Z_{2\mu})J^{\mu}_{FCNC} \tag{8}$$

where ϕ is the mixing angle between the weak eigenstate Z's and the mass eigenstates. Since electroweak precision fits force this angle to be very small[18], we will not include it (although will discuss possible interference terms later). Thus, Z_2 is approximately Z'. Note the fact that if $\sin^2\theta_W$ is greater than 1/4, this breaks down, as discussed above. The current is

$$J^{\mu}_{FCNC} = 2\cos^2\theta_W \overline{q}\gamma^{\mu}P_L q \tag{9}$$

where P_L is the left-handed projection operator. In terms of mass eigenstates, this gives

$$J^{\mu}_{FCNC} = 2\cos^2\theta_W \left(\overline{u}\gamma^{\mu}P_L U_L^{\dagger}\begin{pmatrix}0&0&0\\0&0&0\\0&0&1\end{pmatrix}U_L u + \overline{d}\gamma^{\mu}P_L V_L^{\dagger}\begin{pmatrix}0&0&0\\0&0&0\\0&0&1\end{pmatrix}V_L d\right) \tag{10}$$

where U_L and V_L diagonalize the left-handed $Q = 2/3$ and $Q = -1/3$ quark mass matrices, respectively.

The U_L and V_L matrices are not independent, since one knows that $V_{CKM} = U_L^\dagger V_L$, but the individual values are not known. FCNC processes will then depend on either U_L or V_L matrices alone, and one will not, without further assumptions, know their values.

The papers of Liu[19] and Gomez Dumm et al.[20] calculate the $P - \overline{P}$ mass difference in this model. For Δm_K, for example, they find that

$$\Delta m_K = \frac{2\sqrt{2}}{9} G_F \frac{\cos^4\theta_W}{1 - 4\sin^2\theta_W} |V_{31}^* V_{32}|^2 \eta_Z B_K f_K^2 m_K \left(\frac{M_Z^2}{m_{Z'}^2}\right) \tag{11}$$

Here, η_Z is a QCD correction factor, B_K and f_K are the bag constant and kaon decay constant. Similar expressions can be obtained for other pseudoscalar systems.

Since there is an uncertainty of roughly a factor of two in the Standard Model expression, we assume that the contribution for $K - \overline{K}$ is less than the Standard Model value, and that the $D - \overline{D}$ mixing is less than its experimental limit. (In previous works, similar assumptions were made for the B systems.) For $B - \overline{B}$ mixing, there is very little uncertainty in the hadronic matrix elements, and the primary uncertainty comes from B_B and f_B, which give an uncertainty of approximately 30%; we assume the contribution is less than this uncertainty. For $B_s - \overline{B}_s$ mixing, we require that the contribution be less than 10 picoseconds (for the oscillation time), since that is roughly the current uncertainty. Using updated experimental values, we find the bounds in the first column of Table 1.

One can use these results, as done by Liu[19] to bound the mixing angles. Alternatively, one can assume a Fritzsch-like structure[20], and write (with $i \geq j$) $V_{ij} = \sqrt{m_j/m_i}$ (similarly for U_{ij}), and then find bounds on $m_{Z'}$. Doing so gives an upper bound on $m_{Z'}$, in TeV units, shown also in the first column of Table 1. These bounds, especially for the $B - \overline{B}$ system, are very severe, and are well in excess of the upper bound on the Z' mass. The angles must thus be smaller than one's naive expectation, or the model is excluded. It is also shown by Liu[19]

and Gomez Dumm[20] that if one chose the first or second generation fields to be picked out as being different, then the bound would be much, much stronger, closer to 1000 TeV.

The success of the B-factories has led to stringent bounds on $B \to K f^+ f^-$, $B \to f^+ f^-$ and $B_s \to f^+ f^-$. We now calculate these processes in this model.

For $B \to K f^+ f^-$, only the vector part of the interaction will contribute, and thus the matrix element $\langle K|\overline{s}\gamma^\mu b|B\rangle$ is needed. We use the matrix elements of Isgur, et al. [21], as discussed in Ref. [22], which gives a value of $2f_+ p_K^\mu$, where f_+ is given by $\frac{3\sqrt{2}}{8}\sqrt{\frac{m_b}{m_q}}\exp(\frac{m_K - E_K}{m_K})$. Here, m_q is taken to be a constituent quark mass, or 300 MeV. Given this matrix element, the calculation is straightforward, and we find that the partial width is given, in GeV units, by $\Gamma = 1.7 \times 10^{-15} V_{32}^2 \left(\frac{M_Z}{M_{Z'}}\right)^4$ Using the experimental bound and the Fritzsch ansatz, we find a bound of 1.2 TeV on the mass of the Z', as seen in Table 1. This is substantially weaker than the bound from $B_s - \overline{B}_s$ mixing.

For $B_s \to f^+ f^-$, only the axial vector part of the interaction contributes. Note that a helicity suppression makes the branching ratio proportional to the square of the final state fermion mass. The best experimental bounds are for muon final states ($B_s \to \tau^+\tau^-$ would be very interesting if one could come within a factor of a few hundred of the muonic branching ratio). The standard axial vector matrix element $\langle 0|\overline{s}\gamma^\mu\gamma_5 b|B_s\rangle = f_{B_s} p^\mu$ is used, and we find that

$$\Gamma = \frac{G_F^2 M_Z^4 f_B^2 V_{32}^2 m_B m_\mu^2}{36\pi M_{Z'}^4} \tag{12}$$

Comparing with the experimental bound and using the Fritzsch ansatz gives a lower bound of 0.23 TeV on the Z' mass. For $B \to f^+ f^-$, we find very similar numerical results. Again, this is substantially weaker than the bound from mixing.

It is important to note that even if one abandoned the Fritzsch ansatz (as one must for the model to be phenomenologically acceptable), the bound from quark-antiquark mixing will always be stronger (unless V_{32} is exceptionally small (less than 10^{-3}), in which case the bound

	Model A	Model A'	Model B	Model C
Δm_K	1.6×10^{-4} 4.8 TeV	1.6×10^{-4} 4.8 TeV	4.7×10^{-4} 1.7 TeV	1.7×10^{-4} 4.5 TeV
Δm_D	1.6×10^{-4} 250 GeV	1.6×10^{-4} 250 GeV	4.8×10^{-4} 80 GeV	1.8×10^{-4} 220 GeV
Δm_B	1.4×10^{-4} 30.7 TeV	1.4×10^{-4} 30.7 TeV	4.1×10^{-4} 10.5 TeV	1.5×10^{-4} 28.2 TeV
Δm_{B_s}	1.1×10^{-3} 14.7 TeV	1.1×10^{-3} 14.7 TeV	3.3×10^{-3} 5.0 TeV	1.2×10^{-3} 13.5 TeV
$B_{d,s} \to \mu^+\mu^-$	0.15 230 GeV	0.038 1.0 TeV	0.11 340 GeV	0.32 121 GeV
$B \to K\mu^+\mu^-$	3.2×10^{-2} 1.2 TeV	9×10^{-3} 4.3 TeV	3.5×10^{-2} 1.1 TeV	4.6×10^{-2} 800 GeV

Table 1: Bounds on the models described in the text from several flavor changing neutral processes. The upper number is the bound on $|V_{3i}^* V_{3j}| \frac{m_Z}{m_{Z'}}$, where i and j refer to the relevant quark masses (and the V's are replaced by U's for Δm_D); for the rare B decays, the upper number is the bound on $|V_{3i}^* V_{3j}|^{1/2} \frac{m_Z}{m_{Z'}}$. The lower number is the lower bound on the Z' mass assuming a Fritzsch structure for the V matrix.

on $m_{Z'}$ is less than the direct search bound). In short, there can be *no substantial contribution* to these rare B-decays in this model (since a substantial contribution would lead to an overly large contribution to $B-\overline{B}$ mixing), and this statement is independent of the mixing angles. It should also be noted that we have ignored contributions from Z-exchange and from flavor-changing neutral Higgs exchange. These could destructively interfere, weakening the bounds. However, this would require some fine-tuning and since the Higgs sector has many free parameters, we do not consider this possibility.

In model A', the only difference is in the coupling of the final state leptons to the Z'. While the mass differences are unchanged, there are substantial changes in rare B decays. We find the bounds (see Table 1) on $B \to K\mu^+\mu^-$ to be 4.3 TeV, and the bound from $B_s \to \mu^+\mu^-$ to be 1.0 TeV. Again, the bounds from the mass difference in the $B - \overline{B}$ system are stronger.

We now turn to the $b = 1/2$ models. The embedding of the charge operator now no longer forces $\sin^2\theta_W$ to be less than 1/4, and thus the upper bound on the scale of $SU(3)_L$ breaking no longer applies. As a result, the factors of $1 - 4\sin^2\theta_W$ end up being replaced by $1 - \frac{4}{3}\sin^2\theta_W$.

In Model B, the mass differences in the neutral K, D and B system (but not the B_s) were calculated in Ref. [23], and the bounds from the rare kaon decay $K^+ \to \pi^+ \nu\overline{\nu}$ were calculated[24]. We have reanalyzed these bounds, using updated constraints, and included the bounds from the B_s mass difference, and the rare B and B_s decays discussed above.

Again, if one assumes a Fritzsch-type structure for the U and V matrices, lower bounds on the Z' mass are obtained (one can easily remove that assumption and present results in terms of, for example, the V_{ij} and quark masses). The calculation is the same as for Model A, with different couplings. We find the bounds listed in the third column of Table 1. Again, the bounds from the mass differences are much stronger than from rare B-decays, and are weaker than for Model A (primarily due to the absence of a $1 - 4\sin^2\theta_W$ factor).

In Model C, the only calculation of flavor-changing neutral current effects that we are aware of is the calculation of the mass difference in the neutral kaon system by Ozer, in Ref. [13]. The fourth column of Table 1 lists these bounds. The bounds from mass differences are substantially stronger than in model B.

Models D and E are very different. They are one family models, and thus all generations are treated identically. Due to the existence of isosinglet quarks, there will be flavor changing neutral currents. These models are explicitly explored in Refs. [14] and [15]. FCNC in models with isosinglets have been explored in great detail in a number of papers. The most recent is by Andre and Rosner[25]; the reader is referred to that work and references therein. In most of these works, it is assumed that there is only a singlet isosinglet quark (or if there are more than one, it is assumed that one is much lighter and thus dominates the physical effects), and thus the $Q = -1/3$ mass matrix is 4×4, and it is often assumed that the V_{34} element is the largest. However, the models D and E contain three isosinglet quarks, and if the mass hierarchy of these quarks follows the standard mass hierarchy, the lightest of these will interact much more strongly with the down quark, i.e. the biggest element will be V_{14}. An analysis of the phenomenology of this case would be interesting.

4 Conclusions

$SU(3)_L \times U(1)$ models fall into two categories, depending on the embedding of the charge operator into the $SU(3)_L$ group. The choices of fermion representations further subdivides the models. These models all have tree-level FCNC mediated by gauge bosons. We have calculated the $P - \overline{P}$ mass differences and several rare B decays in these models. In all cases, we find that the contribution from rare B decays is much smaller than those from $B - \overline{B}$ and $B_s - \overline{B}_s$ mass differences, and thus the models explicitly predict that there will be no substantial contribution to these rare B-decays (independent of mixing angles). Lower bounds on gauge boson masses are typically of the order of tens of TeV if one assumes a Fritzch-like structure for the mixing angles. This is a serious problem for the original, minimal model, which has an upper bound of approximately 2-3 TeV for the gauge boson masses. Thus, these models can only survive if the mixing angles are much smaller than one's naive expectation. This would mean that the down-quark mixing matrix would be very nearly diagonal, and thus CKM mixing would have to arise from the $Q = 2/3$ sector. This severely constrains attempts to understand the origin of flavor in these models.

We thank Andrzej Buras for useful discussions. JR would like to thank Colciencias and DIB for financial support and the College of William and Mary for its hospitality. The work of MS was supported by the National Science Foundation grant PHY-023400.

References

[1] P. H. Frampton, Phys. Rev. Lett. **69**, 2889 (1992).

[2] F. Pisano and V. Pleitez, Phys. Rev. D **46**, 410 (1992).

[3] R. D. Peccei and H. R. Quinn, Phys. Rev. Lett. **38** 1440, 1977.

[4] A. G. Dias, C. A. de S. Pires and P. S. Rodrigues da Silva, Phys. Rev. D **68** 115009 (2003).

[5] P. H. Frampton, Proc. of Workshop on Particle Theory and Phenomenology, Ames Iowa (1995) [hep-ph/9507351]

[6] J. T. Liu and D. Ng, Phys. Rev. D **50**, 548 (1994); J. Agrawal, P. H. Frampton and J. T. Liu, Int. J. Mod. Phys. A **11**, 2263 (1996); F. Pisano, Mod. Phys. Lett. A **11**, 2639 (1996); D. Gomez Dumm, Phys. Lett. B **411**, 313 (1997); P. H. Frampton, Int. J. Mod. Phys. A **13**, 2345 (1998) [hep-ph/9711281]; F. Pisano, J. A. Silva-Sobrinho and M. D. Tonasse, Phys. Rev. D **58**, 057703 (1998); N. A. Ky, H. N. Long and D. V. Soa, Phys. Lett. B **486**, 140 (2000); H. N. Long and T. Inami,Phys. Rev. D **61**, 075002 (2000); Y. Kitabayashi and M. Yasue, Nucl. Phys. B **609**, 61 (2001); J. C. Montero, V. Pleitez and M. C. Rodriguez, Phys. Rev. D **65**, 035006 (2002); R. A. Diaz, R. Martinez, J. Mira and J-A. Rodriguez, Phys. Lett. B **552**, 287 (2003); A. Gusso, P. S. Rodrigues da Silva and C. A. de S. Pires, J. Phys. G. **30**, 37 (2004); M. A. Perez, G. Tavares-Velasco and J. J. Toscano, Phys. Rev. D **69**, 115004 (2004).

[7] J. C. Montero, F. Pisano and V. Pleitez, Phys. Rev. D **47**, 2918 (1993).

[8] R. Foot, H. N. Long and T. A. Tran, Phys. Rev. D **50**, R34 (1994); H. N. Long, Phys. Rev. D **53**, 437 (1996); ibid D **54** 4691 (1996); V. Pleitez, Phys. Rev. D **53** 514 (1996).

[9] W. A. Ponce, J. B. Florez and L. A. Sanchez, Int. J. Mod. Phys. A **17**, 643 (2002) [hep-ph/0103100]

[10] W. A. Ponce, Y. Giraldo and L. A. Sanchez, Proceedings of the VIII Mexican Workshop of Particles and Fields, Zacatecas, Mexico, 2001, pp. 341-346 [hep-ph/0201133]; W. A. Ponce, Y. Giraldo and L. A. Sanchez, Phys. Rev. D **67**, 075001 (2003).

[11] D. Ng, Phys. Rev. D **49**, 4805 (1994); A. G. Dias, R. Martinez and V. Pleitez, hep-ph/0407141.

[12] M. B. Tully and G. C. Joshi, Phys. Rev. D **64**, 011301 (2001).

[13] M. Ozer, Phys. Rev. D **54**, 1143 (1996); T. Kitabayashi, Phys. Rev. D **64**, 057301 (2001).

[14] L. A. Sanchez, W. A. Ponce and R. Martinez, Phys. Rev. D **64**, 075013 (2001).[hep-ph/0103244]

[15] R. Martinez, W. A. Ponce and L. A. Sanchez, Phys. Rev. D **65**, 055013 (2002) [hep-ph/0110246]

[16] It is argued in W. A. Ponce, Y. Giraldo and L. A. Sanchez, Phys. Rev. D **67**, 075001 (2003), that two Higgs triplets are sufficient. In this case, the $Q = -1/3$ quarks have zero mass at tree level. It is claimed that a one-loop radiative mass will arise, but one can see (especially by going to a tree-level mass basis) that the masses will only be generated at two-loops. If the b-quark mass is generated only at two-loop order, then the scale of $SU(3)_L$ breaking would be over 500 TeV. It is not clear that such a model is viable–it certainly would suffer from a hierarchy problem.

[17] R. A. Diaz, R. Martinez, and F. Ochoa, Phys. Rev. D **69**, 095009 (2004).

[18] J. T. Liu and D. Ng, Z. Phys. C **62**, 693 (1994).

[19] J. T. Liu, Phys. Rev. D **50**, 542 (1994).

[20] D. Gomez Dumm, F. Pisano and V. Pleitez, Mod. Phys. Lett. A **9**, 1609 (1974).

[21] N. Isgur et al., Phys. Rev. D **39**, 799 (1989).

[22] D. Black, T. Han, H.-J. He and M. Sher, Phys. Rev. D **66**, 053002 (2002).

[23] H. N. Long and V. T. Van, J. Phys. G **25**, 2319 (1999).

[24] N. A. Ky, H. N. Long, L. P. Trung, D. V. Soa and V. T. Van, Talk given at 4th Rencontres du Vietnam, Hanoi, Vietnam, 2000 [hep-ph/0009187]; H. N. Long, L. P. Trung and V. T. Van, J. Exp. Theor. Phys. **92**, 548 (2001) [hep-ph/0104007].

[25] T. C. Andre and J. L. Rosner, Phys. Rev. D **69**, 035009 (2004); J. A. Aguilar-Saavedra, Phys. Rev. D **67**, 035003 (2003).